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Developping an Uncertainty Based Reasoning Approach for Bus Rapid Transit Systems Deployment Planning, Case Study: BRT Lane of Rasht City

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Keywords

Fuzzy logic,
Mamdani inference,
Bus rapid transit,
Public Transportation
Fleet.

Abstract

Today in transportation and engineering applications, there are complex problems associated with uncertainty that are not solvable by conventional computational methods. Bus lanes are among the most well-known and widely used public transportation methods, which are no exception. The purpose of this study is to develop a dynamic basis inference system in order to determine the number of buses of a transit bus lane (case study is a BRT lane of Rasht city) based on the linguistic terms of the experts. For this purpose, a fuzzy inference system is proposed that can model bus dispatch with regard to environmental and climatic conditions, as well as considering inaccuracy and uncertainty and their simultaneous effect. Comparing the proposed method of this study to the current bus deployment systems in the area under study showed that using fuzzy inference system by modeling linguistic terms presented by experts, in addition to reducing waiting time for passengers, has an important role on decreasing fuel consumption and environmental pollution.

1. Introduction

Our real world is too complicated to find an accurate description for all problems [1]. Hard computing approaches can only model relatively simple and accurate systems [2]. Hard computations emphasize accuracy and reliability, while soft computations requires error tolerance and uncertainty in reasoning and decision making [3]. Solutions based on soft computations consist of many computational methods that unlike conventional algorithms, are affected by uncertainty. Soft computational technologies can provide approaches that track and resolve a problem in a changing environment [4]. Public transportation is one of the development indices and among the most important aspects in management of metropolises, which involves numerous dynamic phenomena and problems. This is while bus lanes are among the most famous and widely used public transportation methods. Improvement in the service of these lanes can be done at two levels: First, the improvement of system through changing the bus network in terms of route and number of lanes, and in fact designing a new network, and the other one is to improve the performance of lanes by appropriate and optimal allocation of fleets to the lanes in order to maximize the use of present situation [5]. Launching and implementing public transportation systems costs a lot to the government [6]. Therefore, providing a method to optimize the allocation of existing fleets can be very effective.

Rasht special bus lane system as the area under study normally deploys a fixed number of bus for servicing in each turn,. In this type of servicing, dynamic factors such as demands and also environmental and climate conditions are not taken into consideration. This has led to increased fuel consumption, waiting times and passengers' discontent. The purpose of this study is to model linguistic terminology of traffic and transportation experts to provide a dynamic model for determining the optimal number of transit bus with fuzzy inference. Since many research variables are linguistic and these parameters can be accurately modeled, fuzzy logic is used to model these inaccuracies and ambiguities because fuzzy logic is one of the main components of soft computing and some of its prominent features are using descriptive data for modeling, using viewpoints of experts in modeling and receiving output quantitatively [3]. In this regard, by examining special bus lane of Rasht city and factors such as peak hours, days of the week, rainfall, environment temperature and delay, implemented proposed method will be evaluated and analyzed so that the output, which is the best number of buses per dispatch, will be available to the experts dynamically.

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2. Literature Review

The problem of optimizing bus lanes has been of interest to various researchers for many years and various methods have been proposed for this purpose by these researchers. Cheng and Chung in 1999 created a knowledge-based expertise, which is according to the survey from experts in order to identify uncertainty in demand for buses. A total of 25 fuzzy rules that create and use load factor, load factor variety and average travel speed and also determine the need for extra bus trip, will be used [7]. Lee and Vuchic in 2005 has solved the bus network design in a dynamic and variable demanding manner, with regard to the bust network demand, which depends on the network and frequency of lanes. The objective function of this study is to minimize the total travel time of passengers under limitations of each lane frequency. These authors have simultaneously evaluated the estimate of public transportation demand and optimal network design [8].

Hu and Yang in 2001 presented the optimal method for bus system with objective function of maximizing non-stop passengers using an ant algorithm [9]. Bielli, Caramia and Carotenuto in 2002 in their paper, examined the application of genetic algorithm in optimizing bus network, and presented a new method to calculate genetic algorithm optimization function, which was obtained from the combination with neural network [10]. Yu and Yung in 2005 solved the problem of optimizing bus network in order to minimize the lane change and maximize direct flow of passengers using ant algorithm. According to the authors' claim, a case study of the proposed method in Dalian city has resulted in a reduction in the number of lane change and travel time compared to the present situation [11]. Zhao in 2006 in a study on bus network optimization, placed the objective function to minimize lane change, maximize direct lanes and network coverage. Greedy and tabu search algorithm and a mixture of these two have been used to solve this problem and the best results obtained from the combination of these two algorithms [6]. Sheu in 2005 in order to determine time bus dispatching, conducted a study named "fuzzy clustering method to control the real-time transport of buses" with short-term prediction method of travel demands by using time series prediction model and fuzzy clustering technology in response to the variance of passengers' demand and traffic situation [12].

In most of the previous studies, the aim of researchers for optimizing bus network was related to the network structure and frequency of lanes, timing and also minimize the lane change and maximize direct flow of passengers. As we know change in the existing lanes network for optimization will cost a lot, and this is a shortcoming of these methods. Moreover, optimal allocation of number of buses in the entire network and in each time of dispatching has rarely been observed.

3. Methodology

The proposed method of study is based on the use of fuzzy logic in order to examine the simultaneous effect of time, climate and environmental conditions on the number of buses required to dispatch at various hours of the day in bus rapid transit systems. For this purpose, by modeling the uncertainty of effective input factors (variables such as temperature, rainfall, working and non-working day, peak hours and non-peak hours, as well as the amount of previous service delay) by membership functions (MFs) fuzzy rules have been written in a Mamdani inference engine according to the opinion of experts. The output of the proposed inference system is the number of dispatched buses for each lane in different hours of a day.

3.1. Proposed Fuzzy Model

In terms of philosophy, fuzzy logic states the logical concept in which, unlike Aristotle's logic ruling classical mathematics, it uses multi-value logic to express its concepts. On the contrary to Aristotle's logic which considers everything as existing or non-existing, this logic assumes no boundary between existence and non-existence and anything with a certain degree can belong to a group [13]. Some of the most fuzzy inference system algorithms are Mamdani, Takagi-Sugeno and Tsukamoto inference algorithms. In this study, Mamdani fuzzy inference will be used to design initial fuzzy inference system. In Mamdani fuzzy inference algorithm, the logical results are expressed in a relatively simple structure and compared to Sugeno inference algorithm system which is mostly applicable in designing mathematical and control systems, are used in the area of decision support [14]. Mamdani fuzzy inference model used in this study consists of four steps:

Step 1 (Fuzzification): Involves gaining membership functions and displaying linguistic variables on them. In this step, the membership rate of one element in a set is defined with a value in the range of one (full membership) to zero (no membership). Fuzzy membership functions have different types. In this study, two types of triangular and trapezoidal membership functions have been used.

Step 2 (rule evaluation – inference): In this step of fuzzy modeling, there is no certainty and usually rule evaluation in addition to direct usage of collected data of variables, is done according to the experiences, previous findings, logical reasoning, and experts' opinion.

Steps 3 (aggregation of rule output – composition): After fuzzification of input and output variables, and evaluating rule based on the collected data and previous findings, the combination of charts with respect to the compatibility of rules will be considered. Fuzzy operators including fuzzy AND, fuzzy OR, fuzzy algebraic sum, fuzzy algebraic product and fuzzy operation gamma which will be used to aggregate a set of factors. In this study the fuzzy AND operator has been used. This operator considers the minimum degree of membership and applies on the output membership function.

Step 4 (Defuzzification): The obtained value from inference engine are fuzzy values and they are required to be converted to non-fuzzy values. In this study, centroid method has been used for defuzzification. Centroid is the clear output that is obtained for certain inputs. Figure 1 shows mechanism steps of the proposed fuzzy inference system of the study.

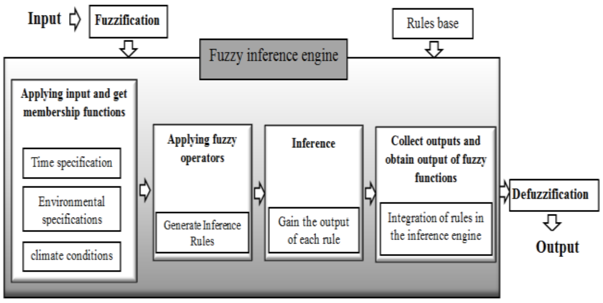


Figure 1. Description of fuzzy inference engine

3.2. Variables Used in the Study

The factors affecting optimal number of bus dispatch which have been investigated together are as follows:

1. Hours of a day
2. Days of a week
3. Rainfall
4. Temperature
5. Delay in bus dispatch

which will be explained in the Table 1.

Table 1. Reaserch variables

Input	Linguistic variable	Membership function and interval	Function Eq.	Descriptions
Hours of the day	Peak 1	Trapezoidal function with interval [6 7 8 9]	$Y = x - 6, \quad 6 < x < 7$ $Y = 1, \quad 7 < x < 8$ $Y = 9 - x, \quad 8 < x < 9$	The time to start schools and offices from 6 to 9 with a peak time between 7 and 8
	Non-peak 1	Trapezoidal function with interval [8 9 11 12]	$Y = x - 8, \quad 8 < x < 9$ $Y = 1, \quad 9 < x < 11$ $Y = 12 - x, \quad 11 < x < 12$	Normal hours of the day
	Peak 2	Trapezoidal function with interval [11 12 14 15]	$Y = x - 11, \quad 11 < x < 12$ $Y = 1, \quad 12 < x < 14$ $Y = 15 - x, 14 < x < 15$	The closing time of schools and offices that reaches its peak hour between 12 and 14
	Non-peak 2	Trapezoidal function with interval [14 15 16 17]	$Y = x - 14, \quad 14 < x < 15$ $Y=1, \quad 15 < x < 16$ $Y = 17 - x, \quad 16 < x < 17$	Normal hours of the day
Hours of the day	Peak 3	Trapezoidal function with interval [16 17 18 19]	$Y = x - 16, \quad 16 < x < 17$ $Y = 1, \quad 17 < x < 18$ $Y = 19 - x, \quad 18 < x < 19$	The closing time of schools and offices at evening shift
	Non-peak 3	Trapezoidal function with interval [18 19 21 22]	$Y = x - 18, \quad 18 < x < 19$ $Y = 1, \quad 19 < x < 21$ $Y = 22 - x, \quad 21 < x < 22$	Normal hours of the day
Days of a week	Working day	Trapezoidal function with interval [0 0 4 6]	$Y = 1, \quad 0 < x < 4$ $Y = 3 - 0.5x, \quad 4 < x < 6$	Weekdays in numerical order
	Non-working day	Triangular function with interval [4 6 6]	$Y = 0.5x - 2, \quad 4 < x < 6$	
Rainfall	Low	Trapezoidal function with interval [0 0 2 4]	$Y = 1, \quad 0 < x < 2$ $Y = 2 - 0.5x, \quad 2 < x < 4$	According to meteorological experts
	Average	Trapezoidal function with interval [3 5 8 10]	$Y = 0.5x - 1, \quad 3 < x < 5$ $Y = 1, \quad 5 < x < 8$ $Y = 5 - 0.5x, 8 < x < 10$	
	High	Trapezoidal function with interval [8 11 30 30]	$Y = 0.35x - 2.6, \quad 8 < x < 11$ $Y = 1, \quad 11 < x < 30$	
Temperature	Very cold	Trapezoidal function with interval [0 0 4 7]	$Y = 1, \quad 0 < x < 4$ $Y = 2.35 - 0.35x, 4 < x < 7$	Temperature in degrees Celsius and classification based on the opinion of the meteorological experts
	Cold	Trapezoidal function with interval [4 7 13 18]	$Y = 0.35x - 1.4, 4 < x < 7$ $Y = 1, \quad 7 < x < 13$ $Y = 3.6 - 0.2x, 13 < x < 18$	
	Mild	Trapezoidal function with interval [13 18 27 32]	$Y = 0.2x - 2.6, 13 < x < 18$ $Y = 1, \quad 18 < x < 27$ $Y = 6.4 - 0.2x, 27 < x < 32$	
	Warm	Trapezoidal function with interval [27 32 50 50]	$Y = 0.2x - 5.4, 27 < x < 32$ $Y = 1, \quad 32 < x < 50$	
Delay in bus dispatch	Low	Trapezoidal function with interval [0 0 2 4]	$Y = 1, \quad 0 < x < 2$ $Y = 2 - 0.5x, \quad 2 < x < 4$	According to the time intervals of bus dispatch and in minute
	Average	Triangular function with interval [3 5 7]	$Y = 0.5x - 1.5, 3 < x < 5$ $Y = 3.5 - 0.5x, 5 < x < 7$	
	High	Trapezoidal function with interval [5 8 15 15]	$Y = 0.35x - 1.6, 5 < x < 8$ $Y = 1, \quad 8 < x < 15$	
The number of optimal bus in each dispatch (output)	Low	Trapezoidal function with interval [0 0 2 4]	$Y = 1, \quad 0 < x < 2$ $Y = 2 - 0.5x, \quad 2 < x < 4$	Based on minimum and maximum demand
	Average	Trapezoidal function with interval [2 4 6 7]	$Y = 0.5x - 1, \quad 2 < x < 4$ $Y = 1, \quad 4 < x < 6$ $Y = 7 - x, \quad 6 < x < 7$	
	High	Trapezoidal function with interval [6 7 8 9]	$Y = x - 6, \quad 6 < x < 7$ $Y = 1, \quad 7 < x < 8$ $Y = 9 - x, \quad 8 < x < 9$	
	Very high	Trapezoidal function with interval [8 10 12 12]	$Y = 0.5x - 4, 8 < x < 10$	

3.2.1. Hours of a Day

Rasht special lane system services passengers from 6 to 22. Time intervals with more demands from passengers can be considered as peak hours including 3 categories. According to Figure 2, peak one, hours of the morning in which offices, organizations, schools and universities begin their activity, with time interval 6 to 9 and peak interval 7 to 8, peak two at the time of closing schools and offices between 11 and 15, with peak interval 12 to 14 and peak three in the evening with time interval 16 to 19 with peak interval 17 to 18. In other hours the demand is reduced to some extent and it is divided into three non-peak intervals of 1, 2 and 3.

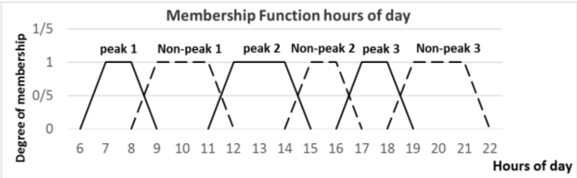


Figure 2. Peak hour membership function

3.2.2. Days of the Week

As shown in Figure 3, days of the week can be divided into working day (non-holiday) and non-working day (holiday). In working days due to the activities of offices, organizations, schools and universities, the number of passengers increases, therefore the service should improve.

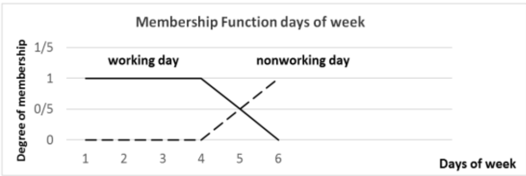


Figure 3. Days of the week membership function

3.2.3. Rainfall

Rainfall considerably affects the demand of bus system, in a way that even in low rainfall, the tendency to use vehicles, even at low distances, and consequently, the need for a bus increases significantly. The rainfall statistic used in this study, which was obtained from meteorological organization of Guilan province, is related to accumulation of rainfall measured by Sardar Jangal station within three hours, and according to the experts' opinion of this organization up to 4mm is low rainfall, from 3 to 10 mm is average rainfall and from 8 to 40 mm is high rainfall, which is indicated in Figure 4.

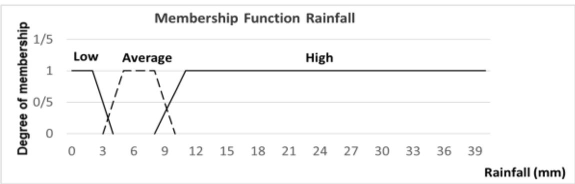


Figure 4. Rainfall membership function

3.2.4. Temperature

The temperature due to intolerance of passengers in hot and cold conditions, can be one of the effective variables in their tendency to use transportation systems. The desirable weather for human is between 20 and 25 degrees Celsius, temperatures higher than that are hot and lower that that are cold, in which inaccuracy of impression from cold and hot weather condition might be modeled in a form of fuzzy variables. As shown in Figure 5, in membership function of this variable, temperatures below zero up 7 degrees are very cold, temperatures between 4 and 18 degree are cold, temperatures between 13 and 32 degrees are mild and temperatures more than 27 to 50 are considered warm. The statistics for this variable, like rainfall, has been obtained from Guilan province meteorological organization.

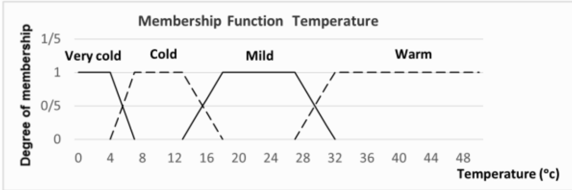


Figure 5. Temperature membership function

3.2.5. Delay in Dispatch

The dispatch of buses must be done from the center according to a regular schedule with a determined interval. In case of delay in dispatch, the waiting time for passengers and their congestion are increased, and in order to solve this problem, the fleet should consider more buses in the next dispatch. For this variable, as shown in Figure 6, we consider three linguistic values. For delays less than 4 minutes with linguistic value "low", delays between 3 and 7 minutes as delay interval "average" and delays between 5 and 15 minutes with delay linguistic value "high" has been shown.

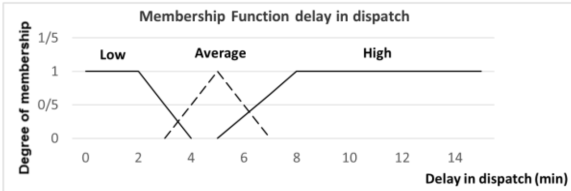


Figure 6. Delay membership function

3.2.6. Output (Number of Buses per Dispatch)

Maximum number of buses considered in each cycle of bus dispatch is twelve (this number has been determined with regard to the maximum need of users of this special lane). Optimizing this number of buses per dispatch, in addition to saving fuel consumption and reducing environmental pollution and costs, improves servicing to the public. The number of bus dispatch based on Figure 7 consists of four "low" linguistic values with 1 to 4 buses, "average" with 2 to 7 buses and "high" with 6 to 9 buses and "Very high" with 8 to 12 buses.

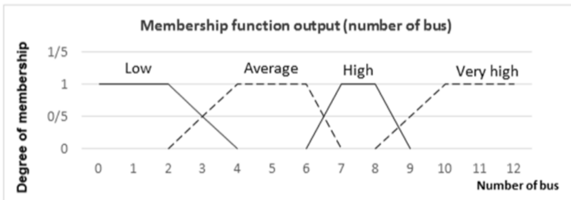


Figure 7. Output membership function

4. Implementation and Discussion

In this section, first the area under study and data will be stated and then the method used to implement and access the research objectives will be explained.

4.1. Data and the Study Area

The area under study is Rasht special lane, which connects the beginning of Imam Khomeini Street with 5 stations to Gil square, is shown in Figure 8. This route is one of the busiest and most commuted streets in Rasht, due to the presence of governmental and nongovernmental organizations as well as schools, markets and business centers. This lane is aimed at reaching a high-speed performance service for the general public and a continuing attempt to reach to bus rapid transit (BRT).

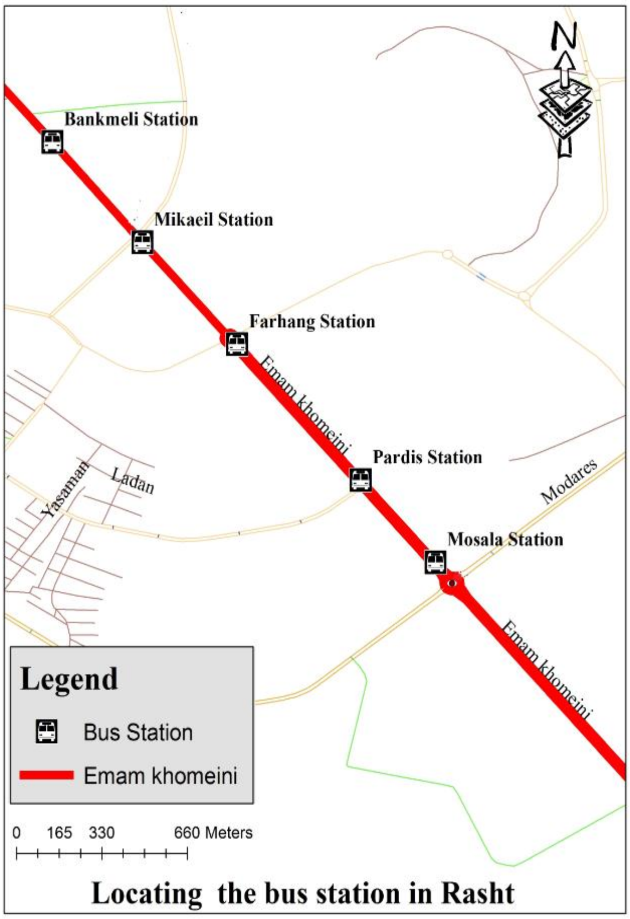


Figure 8. Study Area (Rasht special lane)

4.2. Implementation

After specifying and fuzzifying the input variables referred to in section 3-2, it is necessary to apply the rules. These rules are empirical and involve human and expert opinions in decision making and planning. A sample of rules based on past experiences and findings and logical reasoning is presented in Table 2.

After entering the rules (rule 261 in this study), inference engine of proposed Mamdani method started itswork and according to Figure 9 it combines and aggregates the rules.

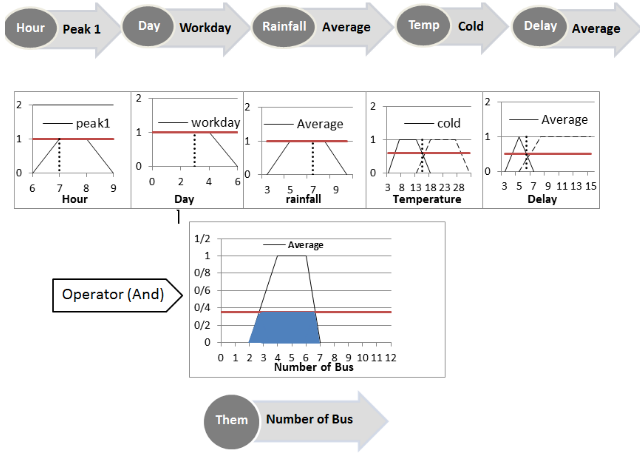


Figure 9. A sample of rules aggregation

One of the outputs raised for problem analysis in this study is display rules and levels in accordance with Figures 10 and 11. Display rules shows a general overview of fuzzy inference process, which in fact with aggregation of rules and then defuzzification from aggregated fuzzy function, can predict the final answer of problem that is the optimal number of buses. For instance, as shown in Figure 10, by considering number 7 for the first variable (peak 1), number 3 for second variable (working day), number 7 for third variable (rainfall), number 15 for forth variable (temperature) and number 6 for fifth variable (delay), the designed system in this study infer that the number 5.72 as the required number of buses to meet the demands, which the result of this inference in order to be used in system must be rounded to a larger integer number. The display levels gives us the two-way connection of input parameters and their impact on the output of problem in 3D, in which a sample of these displays are shown in Figure 11.

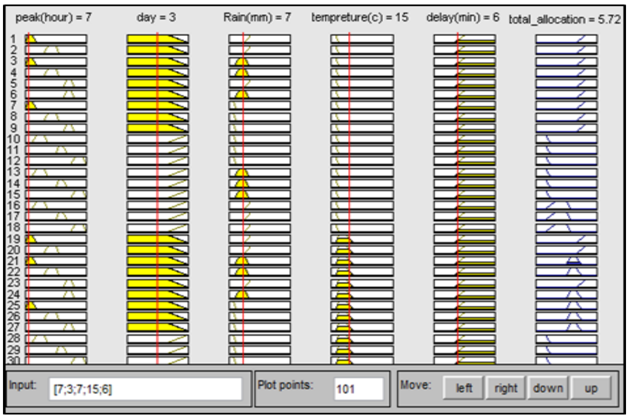
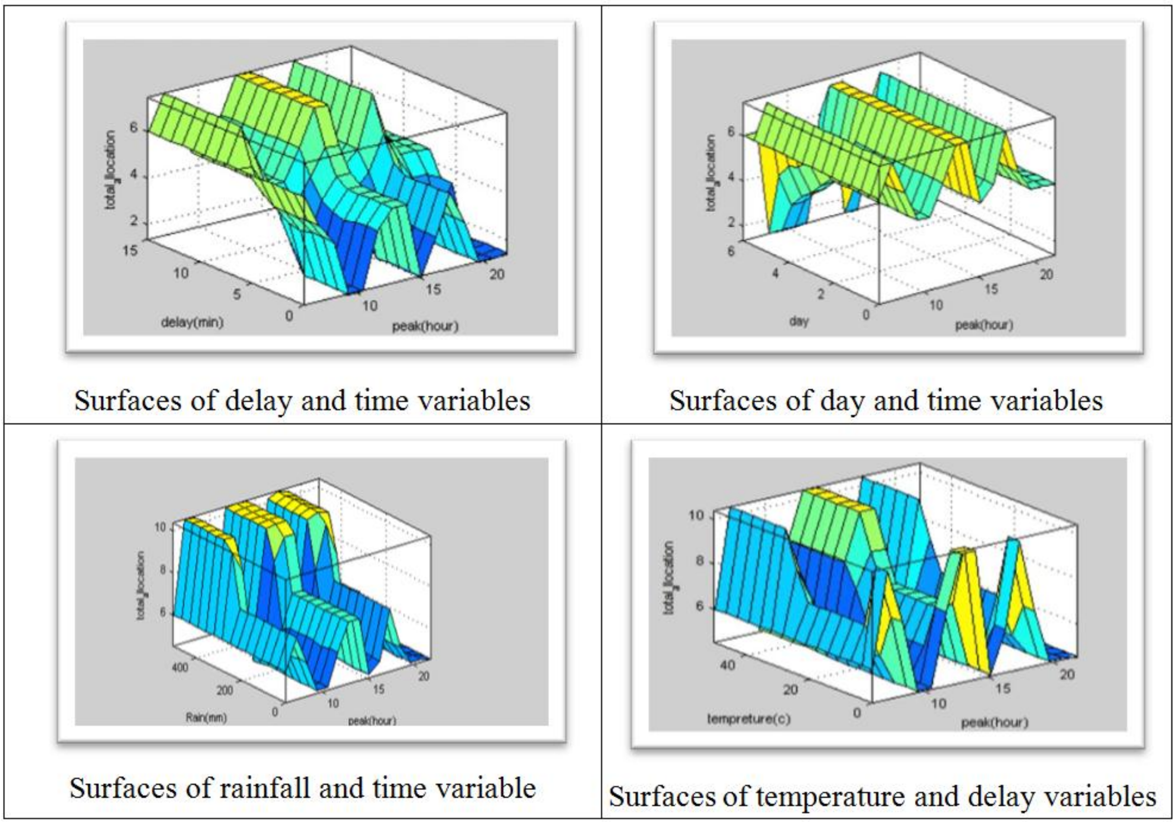


Figure 10. Rules inference

Table 2. Some samples of fuuzy rules in the inference engine of proposed method

		Input										output
		Hour		Day		Rainfall		Temp		Delay		Number of Bus
1	If	Peak1	And	Workday	And	High	And	verycold	And	High	Then	Very high
2	If	Non-peak2	And	Non-workday	And	Low	And	Average	And	Low	Then	Low
3	If	Peak 3	And	Workday	And	Average	And	Cold	And	Low	Then	High
4	If	Peak 1	And	Workday	And	Average	And	Cold	And	Average	Then	Average
5	If	Non-peak1	And	Non-workday	And	low	And	Average	And	low	Then	Low

As shown in Figure 11, due to the computer's inability to display more than three dimensions, the display levels can only show the connection between two variables and their simultaneous impact on the output; Therefore, each time by selecting two input variables and assuming other variables to be constant, we can obtain and analyze the map related to dependency details of selected variables with output.



Surfaces of delay and time variables

Surfaces of day and time variables

Surfaces of rainfall and time variable

Surfaces of temperature and delay variables

Figure 11. Display surface

5. Discussion and Evaluation

According to the all implications and concepts described in previous sections, the proposed method was implemented to provide the bus rapid transit system experts with the output of the best number of buses per dispatch dynamically. In the following, it will be investigated that by using proposed inference engine and determining optimal number of bus dispatch, how factors such as user time, fuel costs, environmental pollution, and heavy traffic in the city will be greatly reduced. Table 3 presents comparative examples of the number of bus dispatch in two strategies of current bus rapid transit dispatch system on the study area and using proposed inference engine.

In the current strategy of bus system, the number of bus dispatch at each time is fixed three buses and this is not related to the hour, working days or holidays, and other environmental and climate conditions. However, with field surveys in different hours of system servicing, it can be concluded that the number of bus dispatched on some days and hours are different from the demands of users in practice, and this demand is dynamic and varies according to different circumstances. The inference system designed in this study involves these circumstances in making decisions for the number of bus dispatched and proposes the optimal number for buses.

For example, in case 1 mentioned in Table 3, the number of bus dispatch according to bus system was three while if timing, environmental and climate conditions and their simultaneous impact on demand are considered, this number reaches eight, that is the number of buses dispatched in practice, do not meet the needs of users and cause congestion of passengers at stations and increase their wasted time. On the other hand, according to sample 3, it is possible to see that the demand is low and therefore a number of empty and with-no-passenger buses that must serve until the end of route. Preventing these events will decrease traffic in streets and as a result reduce fuel consumption and environmental pollution. This is one of the important aspects of smart and environmental-friendly transportation. Optimal management of the transportation system, in addition to improving traffic conditions, prevents the loss of national assets, and as the findings of research and field surveys showed, dynamic information of bus dispatch is one of the effective ways for this optimal management.

Table 3. The comparison of proposed inference system output (the number of optimal buses) with the system used in bus unit

	Date	Day	Hour	Temperature(°c)	Rainfall(mm)	Delay(min)	Number of bus (current strategy of bus system)	Number of bus (proposed inference engine of the study)
1	2017/28/1	Saturday	18:00	4	9	3	3	8
2	2016/16/4	Saturday	12:00	16.4	0.9	6	3	6
3	2017/26/1	Thursday	12:00	17.2	0	0	3	2
4	2016/1/9	Thursday	12:00	30.4	0	2	3	3
5	2016/28/3	Monday	18:00	10.6	13	0	3	5
6	2016/3/4	Sunday	15:00	10.2	0	5	3	2
7	2016/13/5	Friday	21:00	19	0	10	3	2
8	2017/13/2	Monday	12:00	2.2	10	0	3	8

6. Conclusion

The soft computing based approach of transportation applications provide the modeling of dynamic and changing phenomena. In this study since the effects of timing, environmental and climatic conditions on the demand of public transportation system (here, the bus rapid transit system) could not be modeled with certainty and accuracy, a fuzzy inference method was proposed that uses an experienced-based strategy in system operation and converts empirical rules of the skilled operator into an algorithm. The proposed model is able to offer optimal number of bus deploymnet at any time in order to prevent passengers' congestion and fuel loss and also creation of traffic problems at times with no demand.

In order to answer the questions raised in this study, by using a fuzzy logic and its capabilities to human reasoning and flexibility in different conditions in terms of information and input rules, a model was presented that according to transportation and environmental dynamic features, is able to allocate optimal buses per dispatch in transit bus fleet.

7. Acknowledgement

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